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# Non-Doppler Laser Velocimetry: Single Beam Transit-Time L1V

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## Abstract

RECENTLY the theoretical development of a laser-based particle sizing velocimeter (LSV) was presented.[1] The LSV is a non-Doppler laser velocimeter which determines two velocity components and size from scattering signatures of single particles passing through two adjacent, nominally parallel beams. Inherent to the LSV is use of the single beam transit-time laser velocimeter (L1V) concept. A recent paper,[2] a summary of which is presented herein, discussed some of the practical aspects of using the L1V including data acquisition/signal processing and presented some experimental data.

## Contents

### L1V Signal Processing

It has been shown[1,3] that the scattering signature from a particle traversing a single $TEM_{oo}$ mode laser beam can be used to determine the particle speed in the plane normal to the laser beam. This is true for particles significantly (4-10×) smaller than the beam diameter at the exp(−2) intensity points. The time dependence of a scattered light signal $S$ generated by a particle traveling at speed $V_\perp$ in the plane normal to the beam is given by

$$S(t)=S_0\exp(-2(V_\perp(t-t_0)/w)^2) \tag{1}$$

where $w$ is the Gaussian beam radius at the exp(−2) intensity points and time $t_0$ represents the center of the Gaussian peak corresponding to a maximum signal amplitude $S_0$. Here $w$ is known independently and $V_\perp$ is to be determined from $S(t)$.

Analog signal processing methods have been proposed[1,2] and utilized[1,4] but our efforts have been directed toward high-speed analog to digital (A/D) conversion of the signal followed by microprocessor-based digital signal processing (DSP). In the discrete time domain Eq. (1) becomes

$$S_i(t_i)=S_0\exp(-2(t_i-t_0)^2/(w/V_\perp)^2) \tag{2}$$

and the L1V problem becomes one of the estimation of signal parameters $t_0$, $S_0$, and the pulse width $w/V_\perp$ from the discrete samples $S_i$ at times $t_i$. There are several digital signal processing methods applicable to this problem including the discrete Fourier transform (DFT), least-squares curve fitting methods, and other simpler but less accurate estimation/reconstruction algorithms. High-frequency noise on the signal precludes simple digital peak sensing/constant-fraction discrimination algorithms, and this noise can be rejected by least-squares curve fitting or by digital filtering of the DFT spectrum. The following logarithmic least-squares algorithm seems to optimize the tradeoff between processing speed and accuracy.

Taking the natural log of Eq. (2) for linearization purposes

$$\ln S_i=\ln S_0-2(t_i-t_0)^2/(w/V_\perp)^2 \tag{3}$$

and expanding results in the form

$$\ln S_i=a+bt_i+ct_i^2 \tag{4}$$

where

$$a=\ln S_0-2(t_0V_\perp/w)^2 \tag{5}$$

$$b=+4t_0/(V_\perp/w)^2 \tag{6}$$

$$c=-2(V_\perp/w)^2 \tag{7}$$

Minimizing the sum-square error results in the system of normal equations

$$\begin{vmatrix} \Sigma\ln S_i \\ \Sigma t_i\ln S_i \\ \Sigma t_i^2\ln S_i \end{vmatrix} = \begin{vmatrix} n & \Sigma t_i & \Sigma t_i^2 \\ \Sigma t_i & \Sigma t_i^2 & \Sigma t_i^3 \\ \Sigma t_i^2 & \Sigma t_i^3 & \Sigma t_i^4 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} \tag{8}$$

where $n$ is the number of samples. Equation (8) can be solved for $a$, $b$, and $c$ using Kramer's rule and in turn for $V_\perp$, $t_0$, and $S_0$ using Eqs. (5-7). This algorithm has been implemented using a Nicolet digital oscilloscope with sample rates to 2 MHz (12 bits) and to 20 MHz (8 bits) in conjunction with a TI 9900 16-bit microcomputer system. By implementing the $\ln$ operations in a ROM look-up table and calculating and storing the elements and determinant of the 3×3 matrix prior to the experiment, the curve fit and subsequent estimation of $V_\perp$ can be completed using $2n+10$ multiply/divides plus a square root. For reasonable values of the number of samples $n$ per Gaussian peak an L1V throughput rate of 10 kHz is attainable easily with present microelectronic technology. For a noncustom system using for example a stand-alone transient digitizer interfaced to a standard microcomputer (without parallel or multiprocessing) a throughput of more than 1 kHz is possible.

### L1V Sample Volume Characterization

A primary concern for the L1V is the need for accurate characterization of the exp(−2) beam radius $w$ at the optical sample volume. We use a calibrated microscope assembly coupled with a 1000 element linear photodiode array (25 μm centers) mounted at the image plane to perform beam diagnostics. To analyze the beam profile, the peak output values from each diode are digitized and fit to a Gaussian profile using the logarithmic least-squares technique. The curve fitting is performed by the TI 9900 microprocessor system which controls the diode scan, the A/D conversion, transferring the data into microcomputer memory, and, finally, performs the curve fit calculations. Beam diameters are analyzed for several positions along the laser beam ($z$ axis) and a least-squares curve fit of the axial data to the predicted Lorentzian form[5] for $w(z)$ near a diffraction-limited $TEM_{oo}$ laser beam waist is used to estimate $w$ at the waist. The uncertainty in a single measurement of $w$ at one axial location can be 10-20% in worst cases depending on the beam quality,

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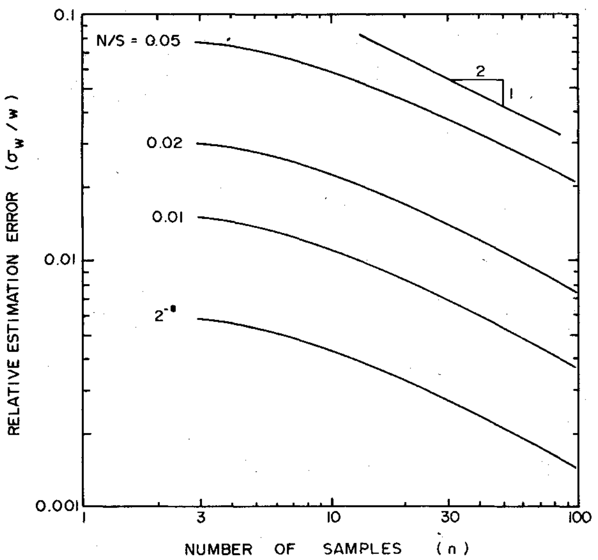


**Fig. 1 Relative error for estimating the Gaussian signal width parameter $w$ from $n$ discrete digital samples (within $1/e^2$ points) with normally distributed random noise contributions. The predictions are for the indicated values of rms noise to signal (amplitude) ratio $N/S$ with least-square fits of Gaussian curves to the data, and assume a 12-bit A/D converter.**

diode-to-diode response variations, and aberrations in the objective system for small beam waists ($\leq 20$ $\mu$m). By measuring the axial $w(z)$ profile this uncertainty can be decreased significantly since the measurement accuracy is better in larger portions of the beam. For $1/e^2$ beam diameters of roughly 50 $\mu$m or greater the technique is accurate to about 1%.

### L1V Accuracy

The tradeoff necessary to have the optical system simplicity of the L1V is potentially decreased accuracy in the velocity measurement. Uncertainties can be introduced through the beam radius $w$ which must be known a priori or through digital signal processing errors. For nonhostile, optically thin flows the uncertainty in knowing $w$ at the sample volume can be made very small. In more difficult measurement environments, for example internal flows in turbulent combustion systems, the value for $w$ clearly will be less certain. In those cases it will probably be necessary to form an "image" of the waist on the output side of a two-ended optical access system to infer properties of the waist in the sample volume using Gaussian beam optics.[5] These uncertainties from $w$ are important but must be considered on a case-by-case basis. The following discussion considers the only DSP errors from the Gaussian curve fit algorithm discussed previously.

The parameters controlling the accuracy of the digital Gaussian curve fit algorithm and, hence, of the L1V velocity measurement include: the number of A/D samples during a Gaussian peak $n$; the ratio of rms noise (high frequency) to the peak amplitude; and the ratio of A/D resolution or digitizing error to the peak amplitude. It is well known from DSP theory[6] that the latter digitizing error is equivalent on an rms basis to $F/(2^b 12^{1/2})$ where $F$ is the full-scale signal value and $b$ the number of bits of A/D resolution. This number is typically 0.11% or less and is, in general, negligible compared to other rms noise contributions.

A computer simulation of L1V signal processing has been used to evaluate the accuracy. Figure 1 indicates predictions of the normalized or relative uncertainty $\sigma_w/w$ for reconstructing a Gaussian peak from discrete digital samples as a function of the number of samples $n$ and the rms noise/signal ratio. The values for $\sigma_w$ were estimated by: 1) assuming a set of Gaussian peak parameters; 2) taking $n$ A/D samples of the assumed signal and perturbing each sample by adding a random noise contribution using a normal distribution random number generator; and, finally, 3) applying the curve fit algorithm to the signal + noise samples. The width parameter of the reconstructed Gaussian less the initially assumed value is then the error in $w$ (and velocity) determination. This process was repeated 1000 times using different random noise perturbations for statistics of the error, and the standard deviation of these errors is reported as $\sigma_w$ in Fig. 1. The mean values of the errors were effectively zero and the data are valid for a 12 bit A/D converter with a full-scale peak signal level.

Figure 1 confirms what one would expect, that the uncertainty in determining $w$ decreases with the number of samples $n$ and increases with the noise level. Three is the minimum number of samples required for the three parameter curve fit. Increasing the number of samples for a given particle velocity involves higher speed A/D conversion and the tradeoffs of cost, fewer bits of A/D resolution, and increased data analysis time. In the limit of large $n$ the slope for any noise level in Fig. 1 approaches $-0.5$ which agrees with classical statistical sampling theory. The acceptable L1V uncertainty would depend on the turbulence intensity levels of interest since data processing errors ultimately appear as velocity broadening. Accuracy comparable to that of LDV seems attainable with the L1V concept.

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